On the Formations of a CubeSat Constellation at the Earth-Moon L1 Libration Point

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- Halo Family Characteristics
- Constellation Placement on Stable Manifold
- Controller and Delta-v Requirements
- Satellite Seperation
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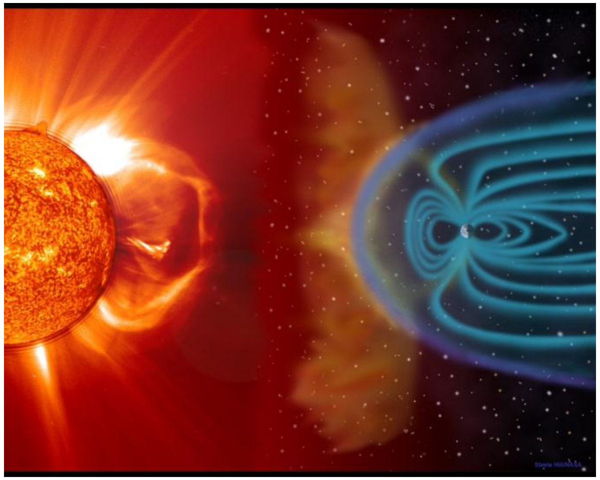
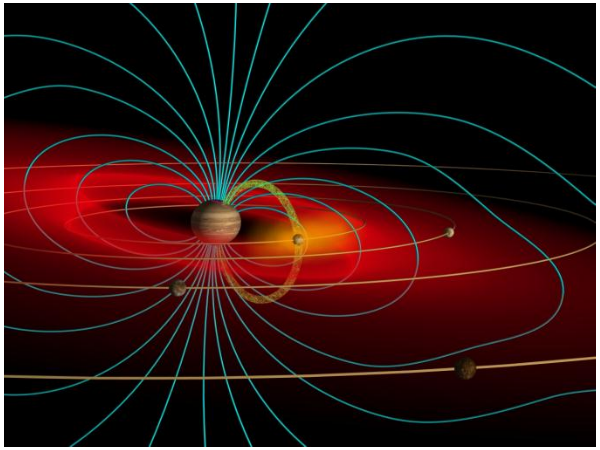
INTRODUCTION

The SOLARA/SARA Mission

- Observe temporal and spatial evolution of solar weather and its interaction with Earth's magnetosphere.
- Produce all-sky map in three bands between 30MHz and 30kHz with spatial resolution of at least 1 arcminute.
- Observe magnetospheric radio emissions from Jupiter, Saturn, Uranus, and Neptune with resolution of 10 arcseconds and search for planetary radio emission at the locations of known giant exoplanets.
- Test the feasibility of a MIMO system in the space environment.
- Demonstrate a communication data rate of at least one order of magnitude higher than traditional (low gain) CubeSat communication systems.





Objective of Project

- Examine various characteristics of halo orbits around the L1 point.
- Simulate the placement of 20 6U CubeSats in orbit around the L1 libration point of the Earth-Moon system.
- Develop station keeping strategy to maintain constellation with as little energy as possible and relative distances of 10km ~100km.

Model:
CR3BP Earth-Moon System

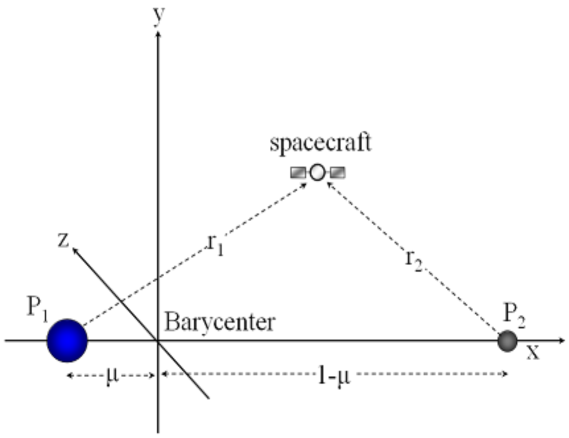
For our study of Halo orbits we consider the nondimensionalized CR3BP (Circular Restricted 3 Body Problem). The basic assumptions are:

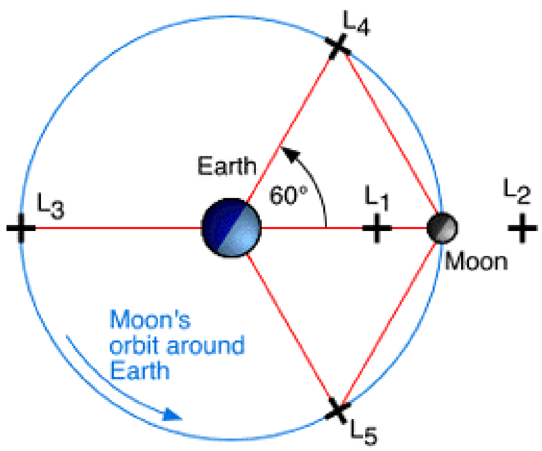
- Moon follows circular orbit
- Mass of third body (i.e. satellite) has negligible gravitational effect.
- Gravitational force between two masses is given by,

$$\vec{F} = \frac{G m_1 m_2}{r^3} \vec{r}$$

Earth-Moon System Libration Points

- Five Libration points exist.
- L1 is of primary interest due to its position do to benefits.



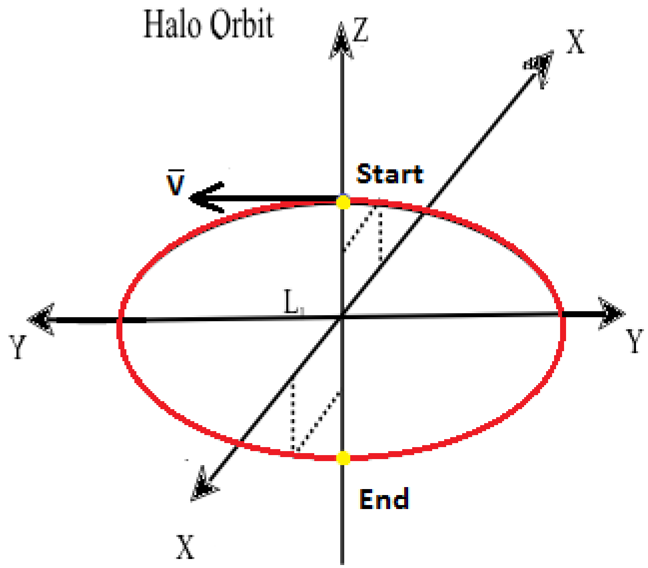


Model: Equations of Motion

Earth and Moon are kept stationary on x-axis using a rotating frame, where (x,y) denotes satellite's position.	$x = X \cdot cos(\frac{2\pi}{T} \cdot t) - Y \cdot sin(\frac{2\pi}{T} \cdot t)$ $y = X \cdot sin(\frac{2\pi}{T} \cdot t) + Y \cdot cos(\frac{2\pi}{T} \cdot t)$
The normalized equations of motion are in the rotated frame are provided by,	$\ddot{x} - 2\dot{y} = -\tilde{U}_x$ $\ddot{y} + 2\dot{x} = -\tilde{U}_y$ $\ddot{z} = -\tilde{U}_z$
Linearization of the CR3BP problem is achieved with,	$\dot{\Phi}(t, t_0) = f'(\vec{x}) \cdot \Phi(t, t_0)$ *where,* $\vec{x}(t) = \Phi(t, t_0)\vec{x}_0$
Numerical solution for the trajectory and STM is computed by solving the first order system of ODEs to the right.	$\dot{\vec{x}}(t) = f(\vec{x}(t))$ $\dot{\Phi}(t, t_0) = f'(\vec{x}) \cdot \Phi(t, t_0)$

NUMERICAL PROCEDURES: CONSTRUCTION OF HALO ORBITS

- The numerical solution for the STM allows for Differential Corrections that can approximate initial conditions leading to a halo orbit.



- A periodic solution must satisfy at the start and midpoint of the orbit,

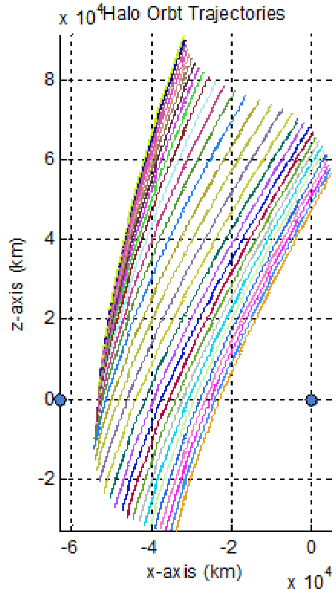
$$p_y = 0$$
$$v_x = 0$$
$$v_z = 0$$

The first is the y position component, and the latter two are the x and z velocity components. It is also worth noting that only half of the solution must be determined.

Description of Halo Orbits

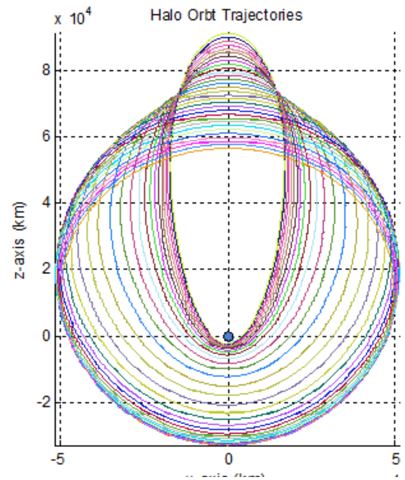
- Larger halo orbits are located near the L1 libration point.
- Orbits closer to the moon are much more elliptical and smaller in their movement relative to the Earth and Moon.
- Largest orbit is approximately 330,000 km in length.

Figure 1



L1 is located at the origin

Figure 2



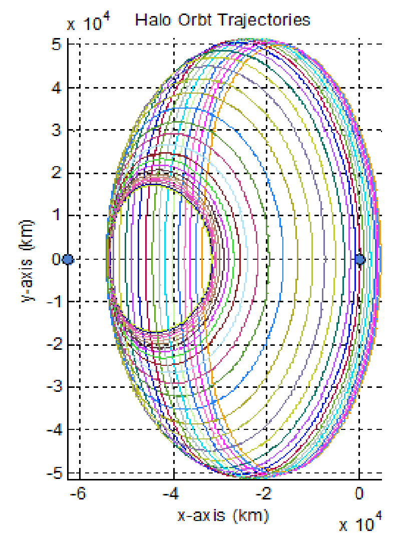
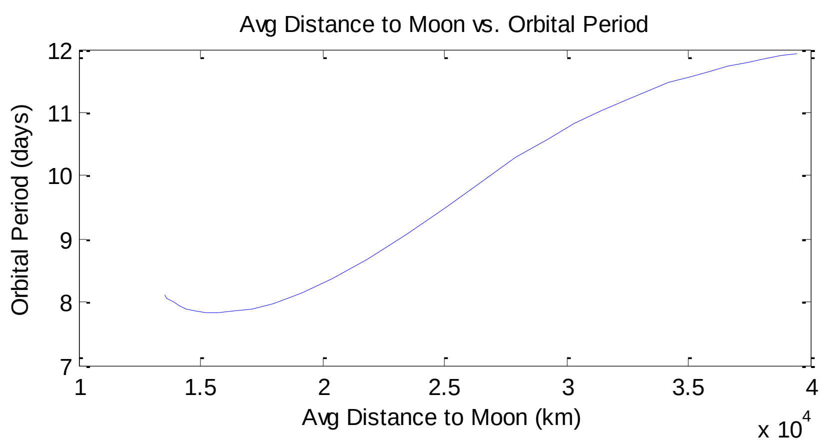
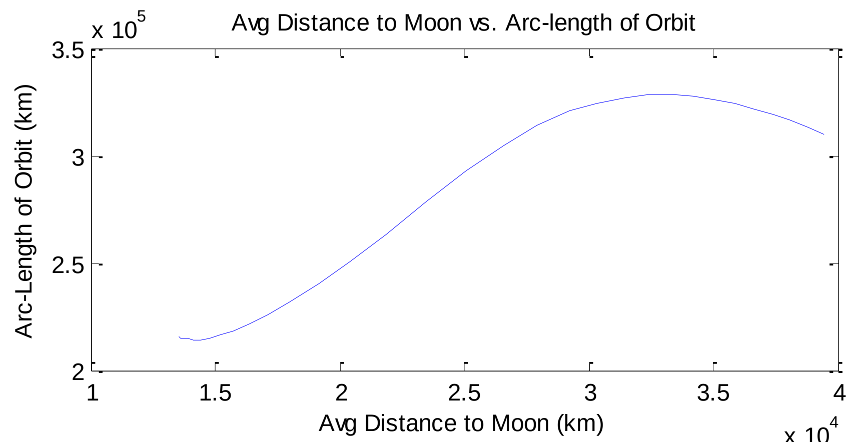
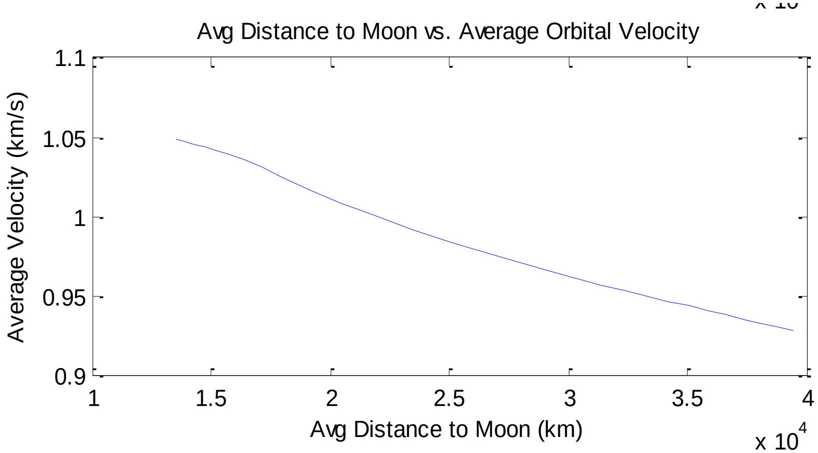


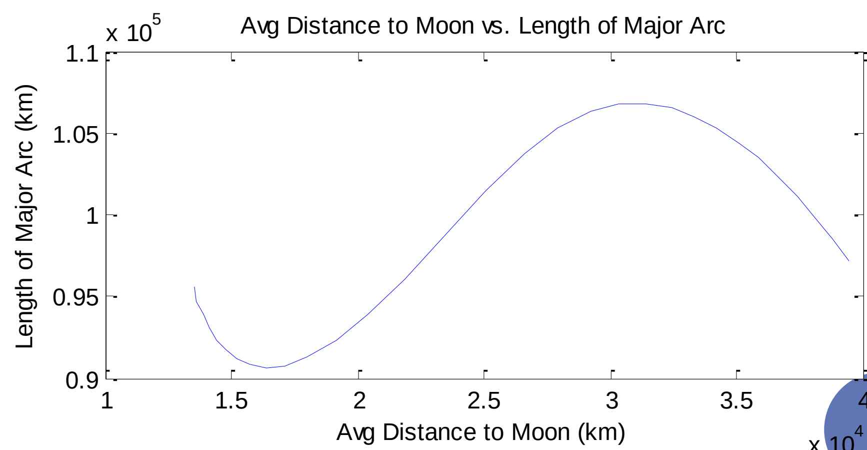
Figure 3

HALO FAMILY CHARACTERISTICS



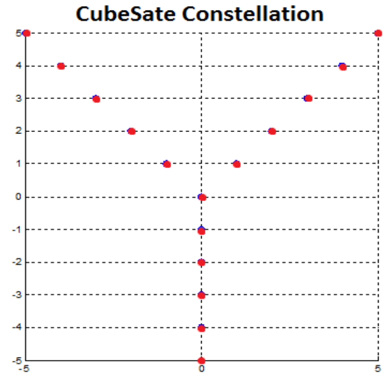




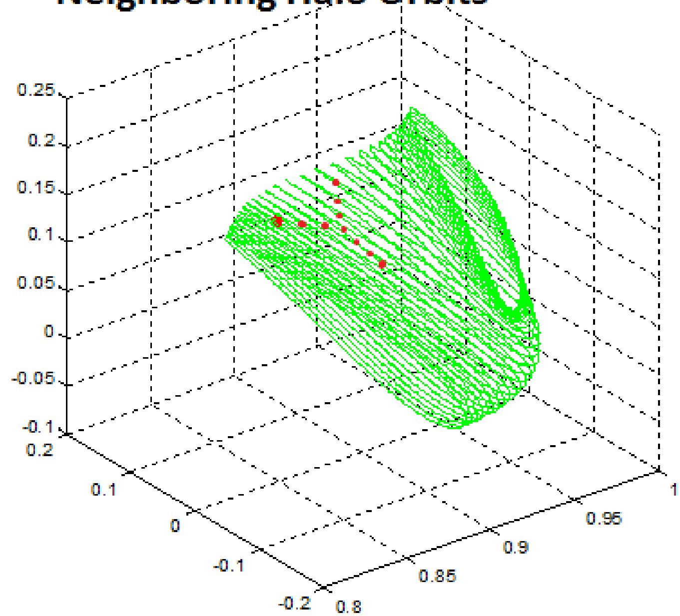


Constellation Placement on Stable Manifold

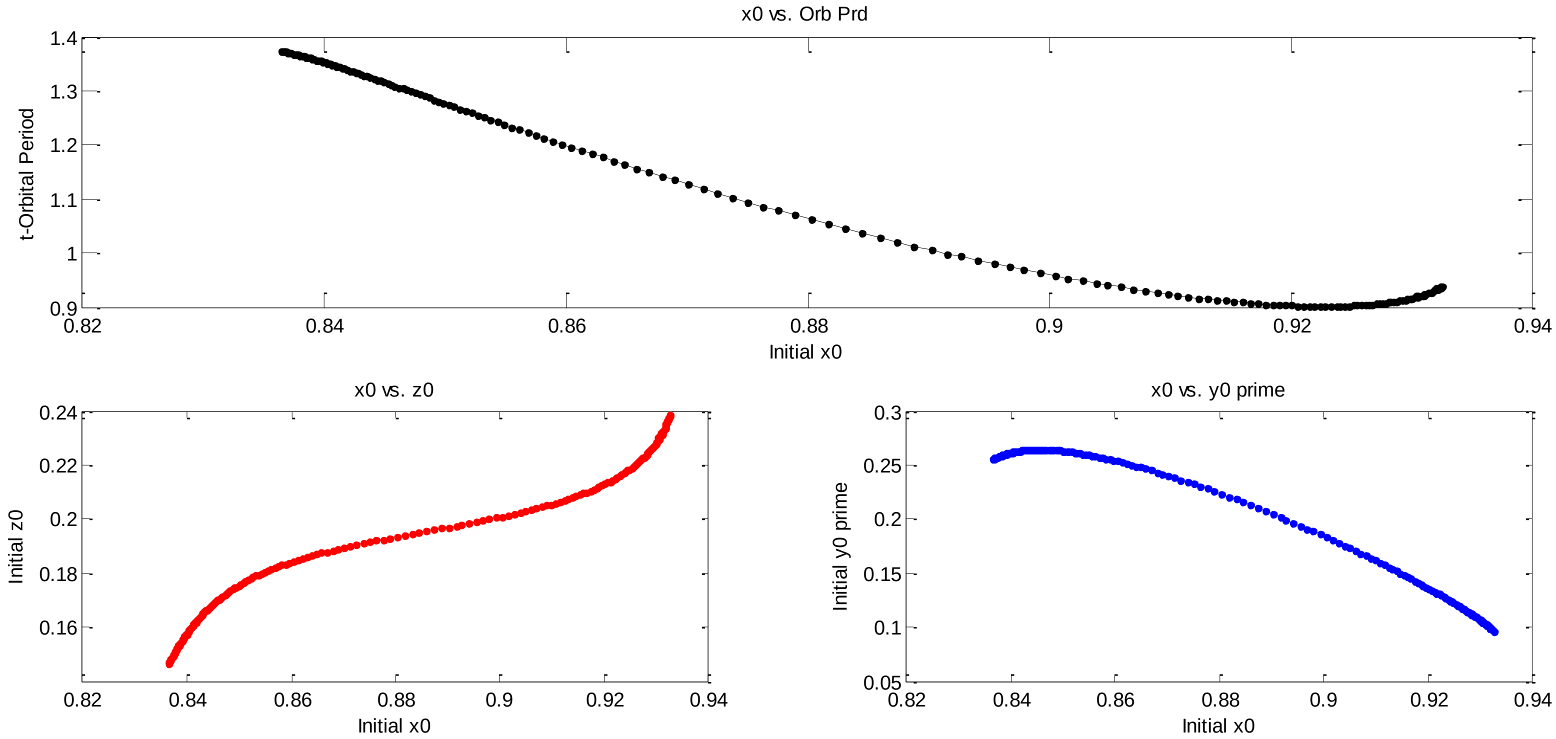
- 20 6U CubeSats forming a Y-shape constellation must be placed in orbit around the L1 libration point and maintain that formation.
- Each satellite must maintain relative distances of 10km~100km with other CubeSats.
- Each region of the stable manifold will impact the feasibility of the two requirements above.
- The selection of appropriate initial conditions will also depend on the characteristics of halo orbits in each region.



Placemen of Constalltion into Neighboring Halo Orbits



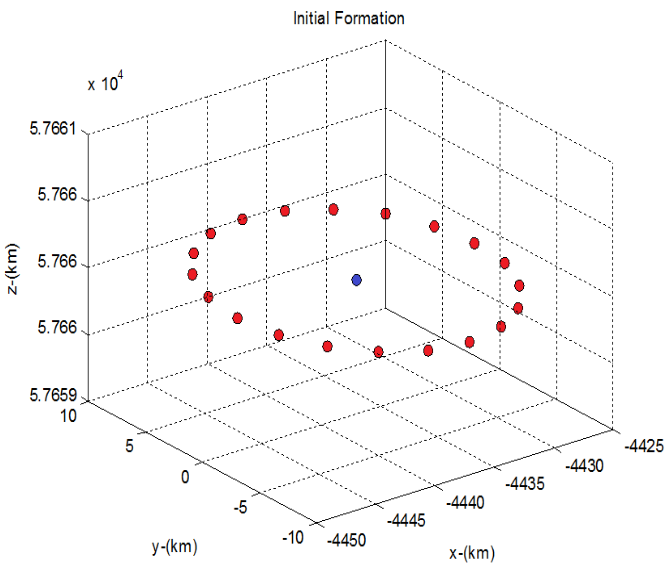
Numerical Procedure for the Construction of Halo Orbits



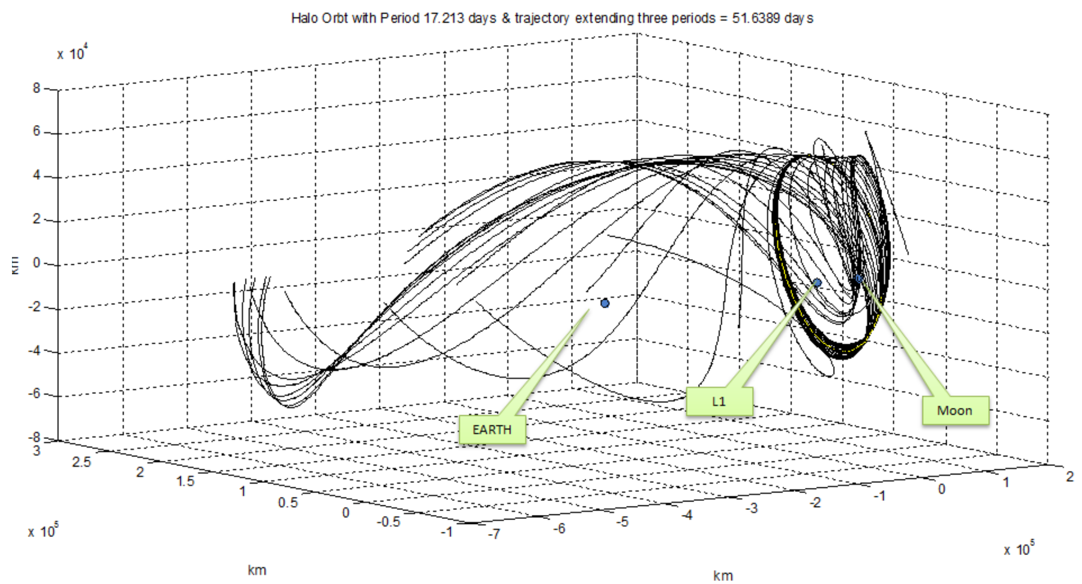
- We can quickly determine a larger range of initial conditions leading to halo orbits around the L1, by interpolating a smaller set of approximated initial conditions.
- Additionally, these curves allow us to examine us the local geometry near the x-z plane to achieve a proper interspacing of the satellites .

Controller and Delta-V Requirements

- Even with good estimates of initial conditions, a stable formation is nearly impossible to maintain.
- The example below computes the trajectories for 20 satellites starting at relative distances of ≤10 km. Within 3-periods, the original formation is completely lost, and their respective distances grow dramatically beyond the desired tolerances.



3-PERIOD TRAJECTORIES (51.64 days) FOR 20 SATELLITES SPACED IN RING FORMATION



Controller and Delta-V Requirements

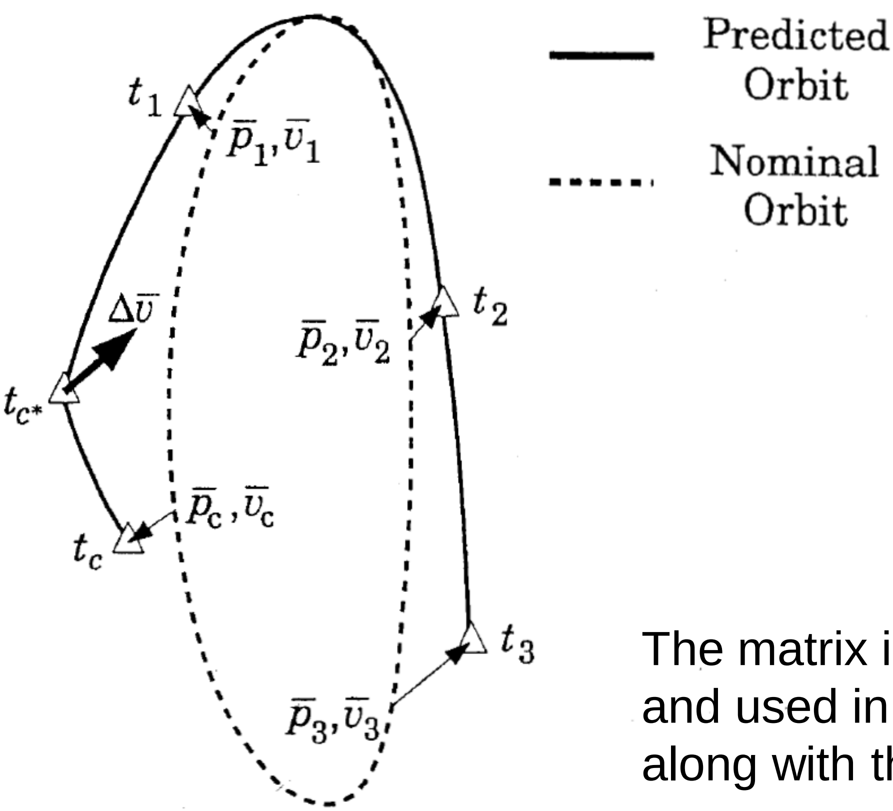
- Objective: Keep each element of the constellation within the vicinity of its original orbit using the least required delta-v. The cost function is given below

$$J = \Delta v^T Q \Delta v + p_1^T R p_1 + v_1^T R_\nu v_1 + p_2^T S p_2 + v_2^T S_\nu v_2 + p_3^T T p_3 + v_3^T T_\nu v_3,$$

- The delta-v corresponding to the relative minimum of the above cost function.

$$\begin{aligned} \Delta v^* \quad = \quad & -\left[Q + B_{10}^T R B_{10} + B_{20}^T S B_{20} + B_{30}^T T B_{30} + D_{10}^T R_\nu D_{10} + D_{20}^T S_\nu D_{20} + D_{30}^T T_\nu D_{30}\right]^{-1} \\ & \times \left[\left(B_{10}^T R B_{10} + B_{20}^T S B_{20} + B_{30}^T T B_{30} + D_{10}^T R_\nu D_{10} + D_{20}^T S_\nu D_{20} + D_{30}^T T_\nu D_{30}\right) v_0 \right. \\ & \left. + \left(B_{10}^T R A_{10} + B_{20}^T S A_{20} + B_{30}^T T A_{30} + D_{10}^T R_\nu C_{10} + D_{20}^T S_\nu C_{20} + D_{30}^T T_\nu C_{30}\right) p_0\right] \end{aligned}$$

CONTROLLER AND DELTA-V REQUIREMENTS

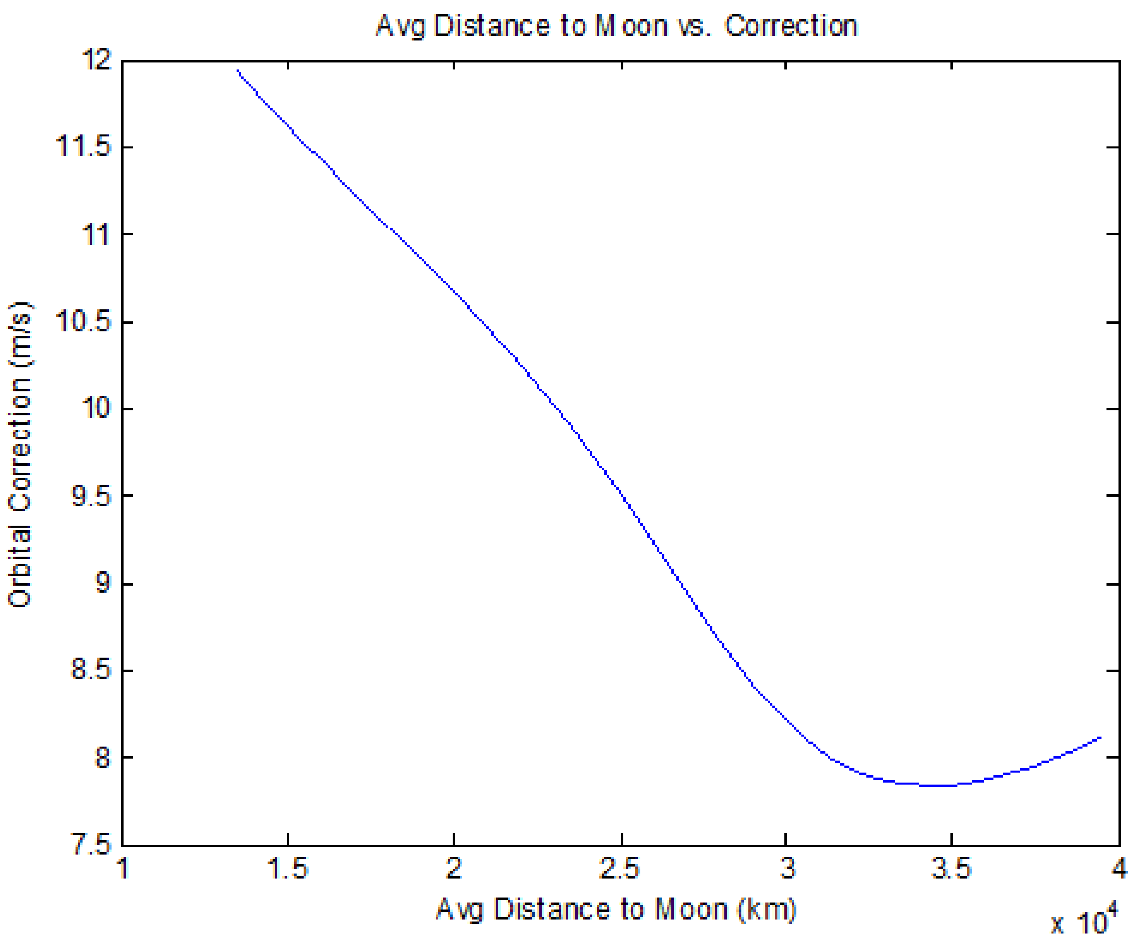


The optimal delta-v that is computed is computed using the STM. By interpolating the components of the matrix, it can be evaluated at arbitrary future time points.

$$\Phi(t_k, t_0) = \begin{bmatrix} A_{k0} & B_{k0} \\ C_{k0} & D_{k0} \end{bmatrix}$$

The matrix is partitioned into components above and used in the expression for the optimal delta-v, along with the weighting matrices.

Controller and Delta-V Requirements

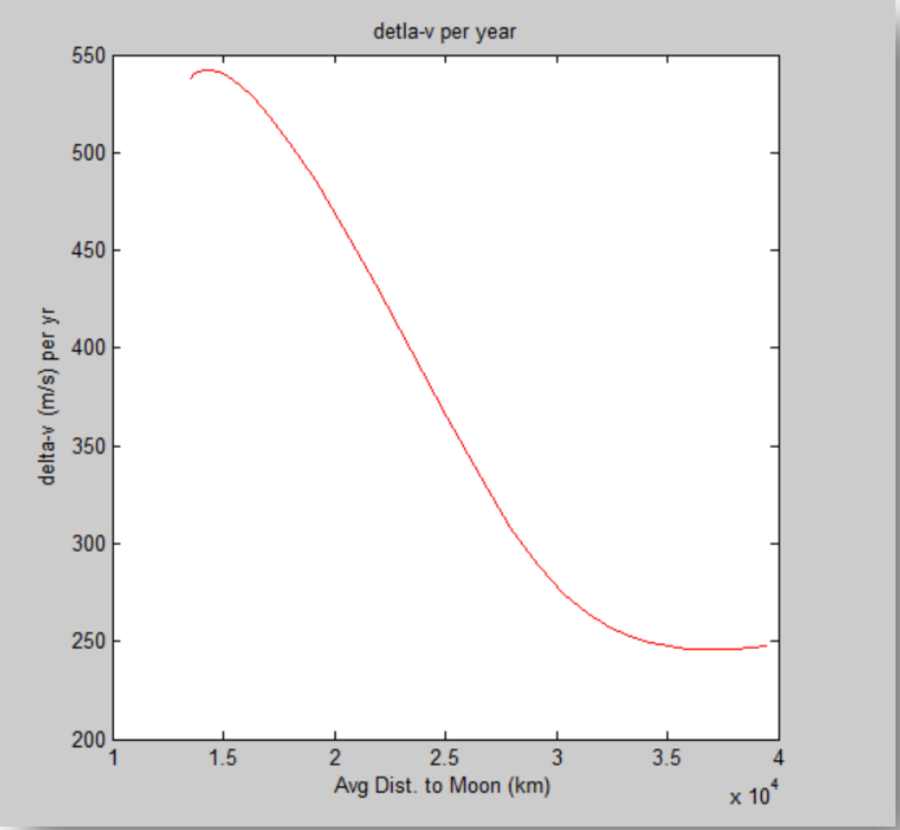


To the left we have a graph demonstrating the delta-v requirements using the Target Point approach for various Halo orbits.

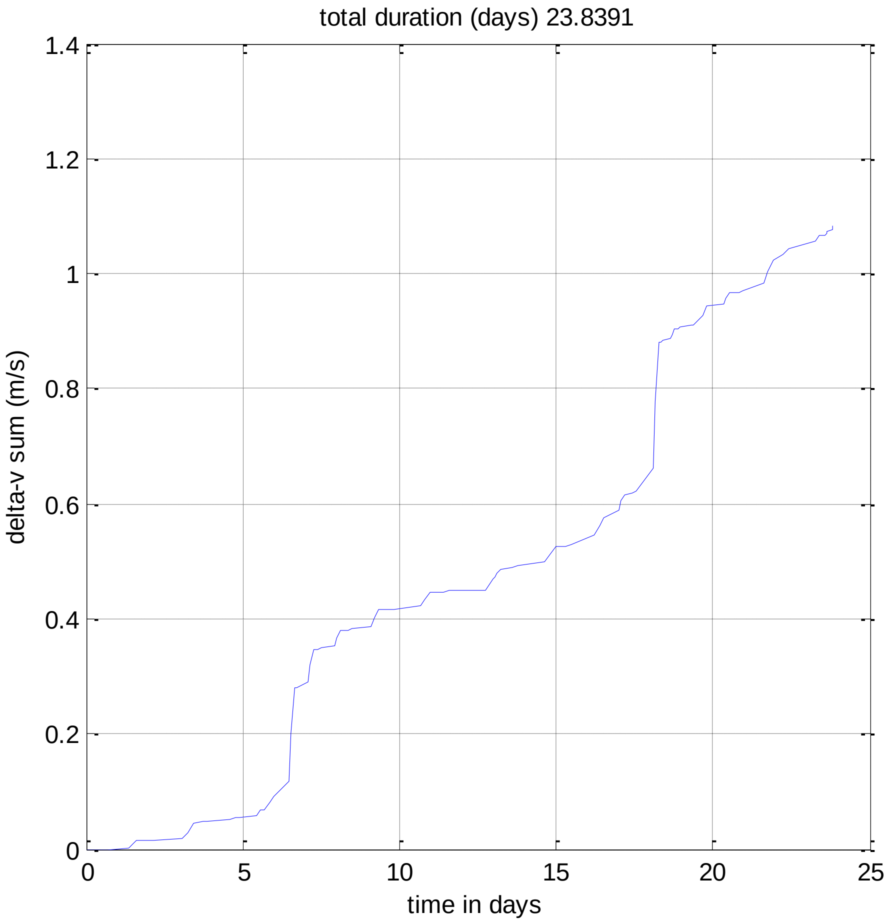
Controller and Delta-V Requirements

•Calculated costs summed over an entire year are significantly high.

•Constraints related to maneuvering timing as well as time between burns quickly add to the costs.



Controller and Delta-V Requirements

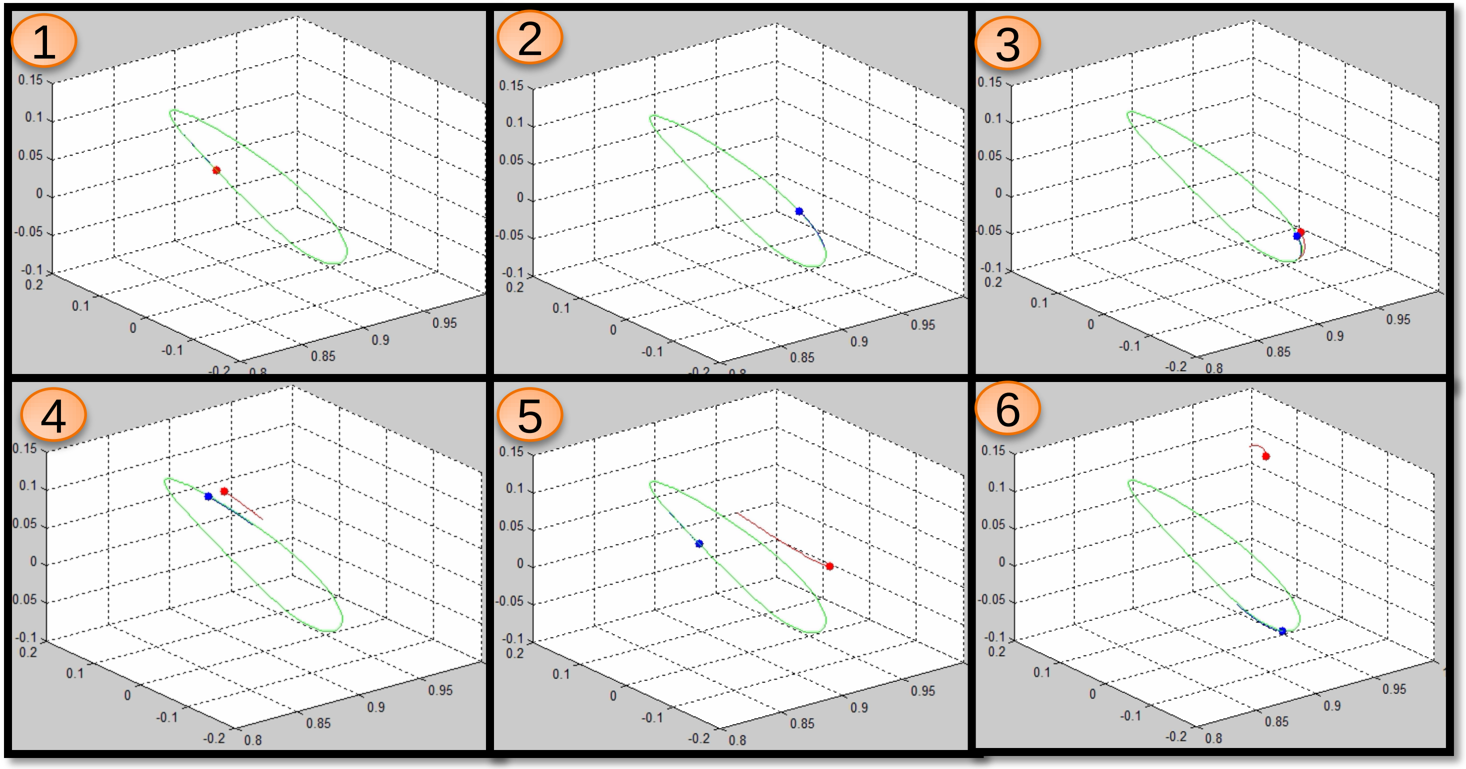


- Better performance is possible using other methods to determine appropriate burn times with delta-v at reduced thrust.

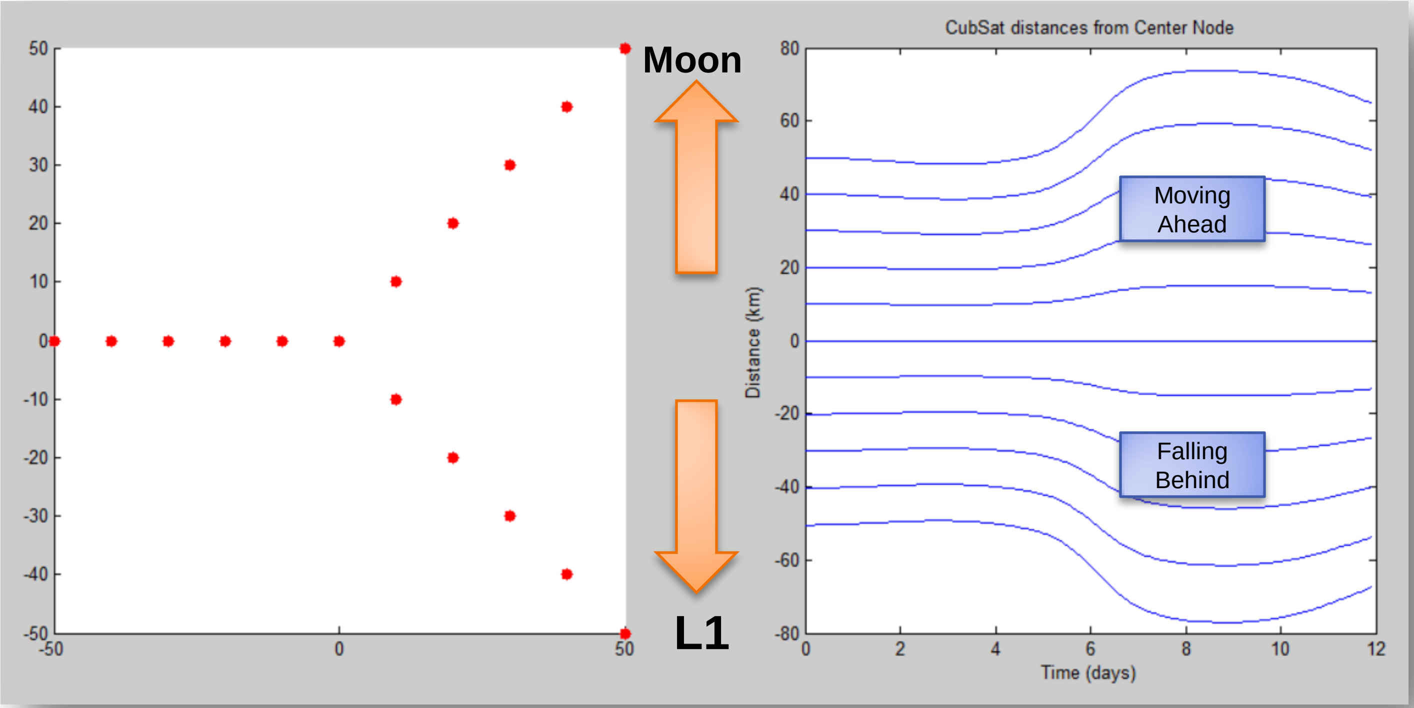
- Floquet Theory offers better performance and control.

CONTROLLER AND DELTA-V REQUIREMENTS

- The simulation presented below demonstrates the performance of the Target Point controller (blue) versus no control (red) for four period (~48 days).



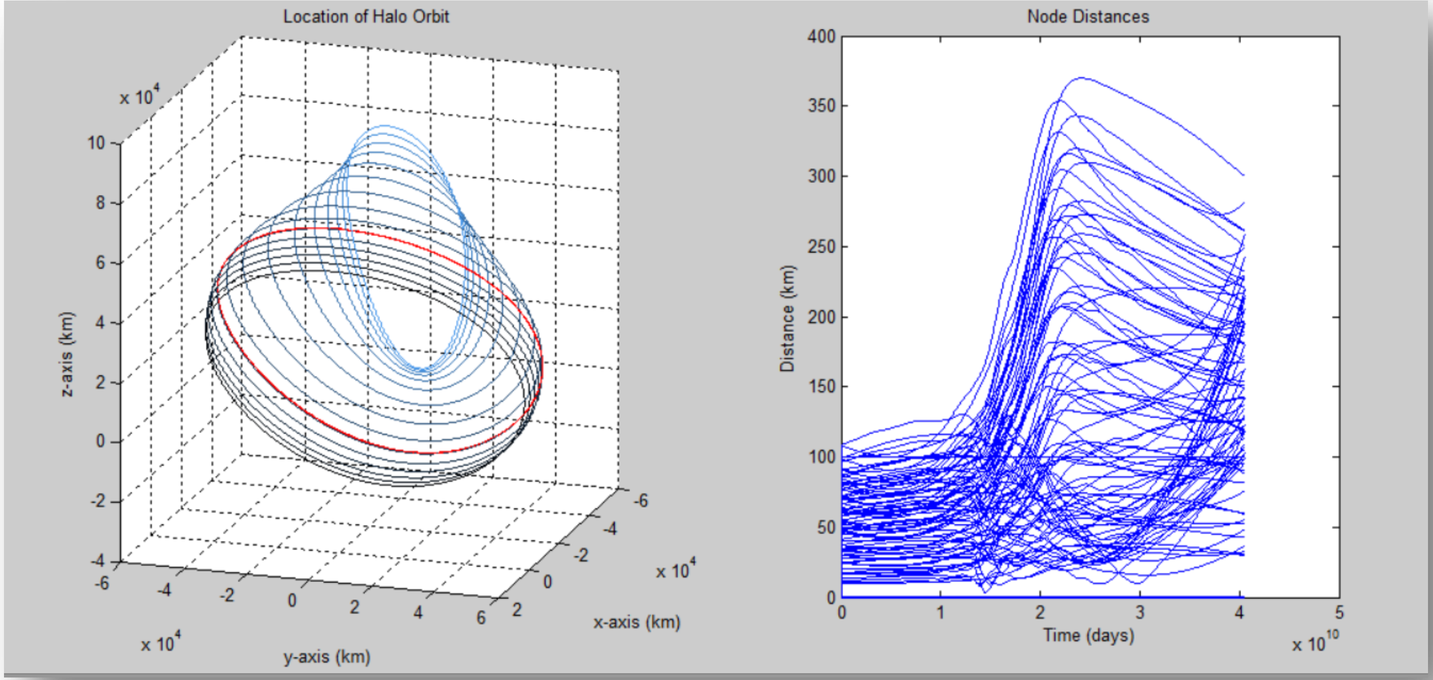
Satellite Seperation



•Along adjacent halo trajectories interspacing of nodes increases due to the variation in orbital periods.

•Additional Station-keeping required to maintain satellite distance within desired ranges.

Satellite Seperation



•Selecting a sample set of trajectories simulated for the desired constellation shows increases in node distances within one period.

•As the regions are selected much closer to the Moon, the satellite distances increase as well.

OTHER AREAS OF INTEREST:

- Station-keeping using the Floquet Mode approach
  - Provides a more efficient station-keeping strategy with over less delta-v required.
  - Characterizes unstable of halo orbits.
  - Does not require use of weighting matrices, which can be difficult to fine-tune when applied with the Target Point Method.
- Explore advantages of continuous controller design.
  - Reduce energy consumption further.
- Incorporate perturbations from other celestial bodies.

References

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2) G. Gomez, K. Howell, K. Howell, C. Simo, and J. Masdemont, "Station-keeping strategies for translunar libration point orbits," in *Proceedings of the AAS/AIAA Space Mechanics Meeting*, Monterey, Calif, USA, 1998.

3) T.M Keeter. Station-keeping strategies for libration point orbits: Target point and flquet mode approaches. Master's thesis, School of Aeronautics and Astronautics, Purdue university, West Lafayette, Indiana, 1994

Questions?